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A Marginal Cost Model of Reinsurance Attachment Points, Catastrophe Risk and Government Intervention

A MARGINAL COST MODEL OF REINSURANCE ATTACHMENT POINTS, CATASTROPHE RISK AND GOVERNMENT INTERVENTION

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SUMMARY

This paper presents a theoretical model of the minimum cost of providing catastrophic insurance coverage. We propose an insurance market model that explains the cost and benefits of reinsurance for catastrophic risk, including the implicit (or explicit) presence of government entities acting as (re)insurers of last resort. Using a two-factor model of reinsurance, we show how reinsurance is optimally layered (with attachment and detachment points) for a given book of business. Our results show that attachment and detachment points are determined optimally by the market participants to minimize the cost of offering insurance protection. Our theory explains why, in a market where a given insurers' cost of bearing risk increases with the size of the risk being insured, the cost of catastrophic insurance can be extraordinarily high and how making the implicit government's guarantee explicit can reduce this cost and increases the policyholders' welfare. Our paper therefore offers public policy implications as to the minimum insurance premium necessary to cover the cost of catastrophic events.

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1. INTRODUCTION

The insurance industry's capacity to absorb large, catastrophic losses is a concern not only for insurance providers, but also for consumers, regulators, and perhaps even more importantly, for public policymakers. Insurers and reinsurers operate efficiently when there are a large number of relatively small, uncorrelated individual risks to insure. When these risks are correlated however, insurers and reinsurers have a more difficult time offering protection as the advantages of pooling diminish. As a result, their cost of capital can become so expensive that insurance is no longer economically sound (see Cummins and Trainor, 2009). Traditionally, reinsurance contracts have been used to share catastrophic risk within the insurance industry. Capital market products such as cat bonds, industry loss warranties, and sidecars have become increasingly popular especially in the higher layers, yet reinsurance remains as the main risk sharing vehicle for catastrophic risk.

Motivation for this paper stems from not only the magnitude and uncertainty regarding potential catastrophic losses, but also from the public policy discussions of the best methods of financing these risks. These discussions include the role of the private insurance market, the role of reinsurers, the role of public financing through government entities (both state and federal level) and the role of capital markets. As meteorologists, oceanographers, wind and structural engineers, and other scientists work toward developing more accurate estimates of expected losses, actuaries, economists, and financial engineers need to work toward developing methods of more efficiently financing these potential losses.

Worldwide, the costs and damage associated with catastrophic events continues to increase (Kunreuther and Michel-Kerjan, 2009). These events can be natural (earthquake, flood, windstorm, etc.) or man-made (terrorism, oil spill). The one source of damage garnering the most interest from the insurance industry is windstorm since flood and earth movement are excluded from most property policies in the US. Since 1990, more than 45% of total catastrophe losses in the US are due to hurricanes and tropical storms (www.iii.org, accessed 11/03/10). The population growth and property

development in coastal areas prone to hurricanes and tropical storms have greatly increased the value of property exposed to loss. In the US alone, hurricane-prone states have more than \$4 trillion dollars in aggregate coastal exposure (AIR Worldwide, 2008).

Significant uncertainty regarding the magnitude of future losses exists. This uncertainty is driven by a lack of understanding of frequency and severity of storms and the potential impacts of global warming trends (Kunreuther and Michel-Kerjan; 2009). Global warming would indicate higher sea surface temperatures, which in turn would indicate an increase in both the frequency and severity of storms (Emanuel, 2007). Additional research, however, has found no impact on the total frequency of storms, but has shown an increase in the frequency of the most severe storms (Elsner et al., 2008). To date, it does not appear that a scientific consensus has been reached on the impact of global warming on storm activity.

Financing of catastrophic risk is increasingly becoming a public policy issue at the state and federal level. The growth of residual markets in hazard prone areas (see Cole et al., 2010; Cole et al, 2009a; Hartwig and Wilkerson, 2007, 2010) increases the importance of finding the proper role and price for private market insurance. Our paper seeks to answer the following four fundamental questions regarding the market for catastrophe insurance. 1- What do insurers bring to the table if it is not capital and underwriting expertise? 2- Where are the optimal attachment and detachment points for reinsurance? 3- When should reinsurance be layered and when should it be proportional? 4- Should the different levels of government be involved in catastrophic risk financing and if so, at what level?

One can divide insurance products into two distinct services: labor and capital. First, insurers provide underwriting and claims adjusting services. Second, insurers have the ability to pool individual risks, diversify risk by line of business and geographically, and attract and supply capital to support the products they sell. Although insurers and reinsurers share common characteristics (e.g. access to the same capital markets, substantial financial distress costs, etc.), the distinctions between these two services are especially relevant to the provision of catastrophe insurance. The cost of capital and the ability to underwrite and adjust claims at the individual risk level are critical factors in determining where in the loss distribution (primary layer, working layer, excess layers, etc.) an (re)insurer would be most efficient in providing coverage. Reinsurers often have better diversification opportunities than primary insurers if only because they do not face the same regulatory oversight as a primary insurer.

Also, reinsurers are generally larger global entities (see Table 1) than primary insurers, which allows them to gain access to a much wider set of potential sources of risk whose losses are presumably less correlated, thus increasing their potential for diversifying their losses.

Segment	Consolidated	Reinsurance	PP auto & Home	Comm. Property	Comm. Auto	Comm. Casualty	A&H	Credit	Fin. Guar.	Med. Mal.	Fidelity
Number of Companies	2402	67	295	114	65	552	9	33	18	135	63
Average Surplus (\$ 000's)	197,827	1,155,440	318,485	189,351	49,138	278,077	55,091	103,956	362,184	75,714	70,110
% of Cons. surplus	100%	16.29%	19.77%	4.54%	0.67%	32.30%	0.10%	0.72%	1.37%	2.15%	0.93%

Line contributions do not add to 100% because of omitted insurance lines, such as Workers' Compensation.
Source: Bests' Aggregates and Averages.

Table 1 shows that US reinsurers have substantially more surplus per company (more than 300% larger than the next largest segment) than US primary insurers in any other segment. US reinsurers hold more than 16% of the total industry's surplus, which makes the US reinsurance industry as a whole the third largest holder of surplus in the US insurance industry behind commercial casualty insurers (32% of total surplus) and personal automobile and homeowner insurers (20% of total surplus).¹

Because of the sheer size of each reinsurer, we should expect the reinsurance industry's bankruptcy costs to be lower in expectation than a primary insurer operating in the same layer; this means that the reinsurance industry's marginal cost of capital should be lower.² Looking at the three largest lines based on surplus (these three lines account for close to 70% of the total consolidated surplus) we should expect reinsurers to have the lowest cost, followed by personal insurers and commercial casualty insurers. Using the capital-to-premium ratio³ as our measure of capital cost, Table 2 provides evidence

¹ The importance of reinsurers' surplus would have been much larger had we taken an earlier year since 2008 was a bad year for reinsurers.

² It is important to remember that primary insurers and reinsurers are not offering services in the same layers; there is nothing in the capital-to-premium ratio that accounts for what would have been the cost of capital of the primary insurers had they insured large, infrequent (i.e., catastrophe) events, or what would have been the reinsurers' cost of capital had it been forced to insure the first dollar.

³ Zanjani (2002) uses the adjusted capital-to-premium ratio, where net income and surplus are discounted based on the segment's payout tail. Panel A of Table 2 is most similar to Zanjani's calculation, less the discounting.

that reinsurers have the lowest cost over the past ten years, which is contrary to Zanjani (2002). The only difference between panels A and B in Tables 2 and 3 is that Panel A is the average of ratios whereas Panel B is the ratios of the averages.

Table 2. Capital-to-premium (or capital cost) ratios of selected property and casualty insurance segments											
Panel A. Capital cost ratios by year (1999-2008) and average of capital cost ratios											
Fiscal year end	Consolidated	Reinsurance	PP auto & Home	Comm. Property	Comm. Auto	Comm. Casualty	A&H	Credit	Fin. Guar.	Med. Mal.	Fidelity
2008	-7.0%	-13.7%	-3.9%	-1.0%	-1.0%	1.3%	26.1%	-4.9%	-373.1%	14.4%	26.3%
2007	12.2%	21.1%	8.6%	17.5%	7.0%	14.1%	14.6%	16.1%	-40.0%	23.9%	22.8%
2006	17.2%	35.7%	13.1%	17.9%	7.6%	24.3%	5.8%	14.9%	106.8%	20.0%	18.3%
2005	10.6%	4.0%	9.0%	18.5%	5.3%	7.6%	37.9%	14.0%	89.8%	9.6%	17.9%
2004	9.8%	11.0%	10.0%	12.5%	5.0%	6.9%	12.5%	10.6%	78.2%	-1.3%	15.4%
2003	9.6%	20.3%	7.6%	15.4%	8.4%	4.7%	9.4%	16.1%	72.0%	-4.3%	10.6%
2002	-5.0%	-12.1%	-6.5%	4.6%	7.9%	-4.9%	6.3%	10.7%	66.9%	-25.5%	1.9%
2001	-10.1%	-30.1%	-11.1%	-9.7%	9.4%	-9.1%	4.2%	4.0%	77.5%	-17.9%	3.0%
2000	-2.6%	-5.4%	-6.3%	-0.5%	11.0%	0.3%	1.1%	5.5%	92.6%	-7.4%	4.0%
1999	2.5%	-9.8%	2.8%	-9.7%	3.1%	2.7%	-7.3%	6.3%	103.4%	1.0%	9.6%
10-year average	3.7%	2.1%	2.3%	6.6%	6.4%	4.8%	11.1%	9.3%	27.4%	1.3%	13.0%

The Capital Cost Ratio is calculated as (Net Income + Unrealized Capital Gains + Income Taxes – Investment Income) / (Direct Premium Written + Policyholder Dividends – Investment Income); Investment Income is calculated as Return on Investment * Surplus.
Source: Bests' Aggregates and Averages.

Panel B. Capital cost ratio averages (1999-2008)											
Year	Consolidated	Reinsurance	PP auto & Home	Comm. Property	Comm. Auto	Comm. Casualty	A&H	Credit	Fin. Guar. (ex 2008)	Med. Mal.	Fidelity
10-year average	4.0%	1.0%	2.6%	9.3%	5.4%	5.6%	5.4%	8.3%	64.3%	3.1%	16.1%

The Average Capital Cost Ratio is calculated as (10 year Total Net Income + 10 year Total Unrealized Capital Gains + 10 year Total Income Taxes – 10 year Total Investment Income) / (10 year Total Direct Premium Written + 10 year Total Policyholder Dividends – 10 year Total Investment Income); Investment Income is calculated as Return on Investment * Surplus.
Source: Bests' Aggregates and Averages.

The capital-to-premium ratio does not take into account the insurers'/reinsurers' expenses. Accounting for underwriting expenses (that are much larger for primary insurers than reinsurers), the capital-to-premium-net-of-expense ratio is larger for primary insurers than for reinsurers. This implies that the reinsurers' cost of assuming additional risk is lower than the cost to primary insurers. When expenses

are taken into account, Table 3 shows again that reinsurers have on average a lower capital cost. Clearly, including expenses in the capital-to-premium ratio calculations increases the capital costs for all segments, but less so for reinsurance than for other segments, as we anticipated.

Table 3. Capital-to-premium-net-of-expenses ratio of selected P&C insurance segment

Panel A. Capital-to-premium-net-of-expenses ratio by year (1999-2008), average of ratios, and impact of netting loss adjustment expenses (LAE) and underwriting expenses compared to capital-to-premium ratio (table 2, panel A).

Year	Consolidated	Reinsurance	PP auto & Home	Comm. Property	Comm. Auto	Comm. Casualty	A&H	Credit	Fin. Guar	Med. Mal.	Fidelity
2008	-10.8%	-17.6%	-6.8%	-1.2%	-1.3%	2.1%	34.2%	-6.8%	-532.9%	22.9%	40.9%
2007	19.4%	30.4%	13.9%	24.0%	9.6%	22.1%	17.6%	25.3%	-51.4%	41.6%	34.3%
2006	27.4%	51.1%	21.4%	24.8%	10.1%	37.7%	23.0%	22.8%	162.5%	35.8%	27.5%
2005	9.2%	-9.9%	9.4%	21.7%	4.0%	7.0%	22.8%	15.5%	110.2%	12.9%	22.9%
2004	15.3%	15.0%	16.1%	18.6%	6.6%	10.6%	14.1%	16.9%	99.5%	-2.2%	21.6%
2003	15.0%	27.6%	12.0%	22.7%	11.1%	7.1%	10.7%	23.9%	84.6%	-7.9%	16.8%
2002	-7.6%	-15.8%	-10.1%	5.5%	11.2%	-7.4%	7.3%	16.1%	79.3%	-45.8%	3.3%
2001	-16.1%	-42.2%	-17.9%	-12.4%	13.7%	-13.8%	5.0%	6.3%	94.4%	-34.1%	7.3%
2000	-4.4%	-8.2%	-10.7%	-0.6%	16.5%	0.4%	1.3%	8.6%	142.7%	-17.4%	9.7%
1999	4.3%	-16.0%	5.0%	-13.2%	4.8%	4.4%	-9.0%	10.2%	144.9%	2.4%	25.8%
10-year average	5.2%	1.5%	3.2%	9.0%	8.6%	7.0%	12.7%	13.9%	33.4%	0.8%	21.0%
Expenses' impact	+1.8%	+0.5%	+1.2%	+2.8%	+2.5%	+2.6%	+3.4%	+4.9%	+6.2%	-0.2%	+8.3%

The Capital-to-Premium-Net-of-Expense Ratio is calculated as (Net Income + Unrealized Capital Gains + Income Taxes – Investment Income) / (Direct Premium Written + Policyholder Dividends – LAE – Underwriting Expenses – Investment Income); Investment Income is calculated as Return on Investment * Surplus. The Expenses' impact is calculated at the 10 year average in Table 3 Panel A minus the 10 year average in Table 2 Panel A.
Source: Bests' Aggregates and Averages.

Panel B. Capital-to-premium-net-of-expenses ratio averages (1999-2008) and impact of netting loss adjustment expenses (LAE) and underwriting expenses compared to capital-to-premium ratio (table 2, panel B).

Year	Consolidated	Reinsurance	PP auto & Home	Comm. Property	Comm. Auto	Comm. Casualty	A&H	Credit	Fin. Guar. (ex 2008)	Med. Mal.	Fidelity
10-year average	6.2%	1.4%	4.3%	12.5%	7.5%	8.7%	6.5%	12.6%	82.8%	5.6%	25.4%
Expenses' impact	+2.2%	+0.4%	+1.7%	+3.2%	+2.1%	+3.1%	+1.0%	+4.3%	+18.5%	+2.5%	+9.3%

The Capital-to-Premium-Net of Expenses Ratio is calculated as (10 year Total Net Income + 10 year Total Unrealized Capital Gains + 10 year Total Income Taxes – 10 year Total Investment Income) / (10 year Total Direct Premium Written + 10 year Total Policyholder Dividends – 10 year LAE Expenses – 10 year Underwriting Expenses – 10 year Total Investment Income); Investment Income is calculated as Return on Investment * Surplus. The Expenses' impact is calculated at the 10 year average in Table 3 Panel B minus the 10 year average in Table 2 Panel B.
Source: Bests' Aggregates and Averages.

Leveraging this advantage in marginal cost by providing insurance policies directly to policyholders would reduce insurance costs if reinsurers were well informed about the quality of the risks being assumed. However, reinsurers do not have the same level of investment that primary insurers have made in underwriting and claims adjusting services. This investment allows primary insurers to offer these services at a lower marginal cost than could be provided by reinsurers. The result is that reinsurers have an underwriting ability at the individual risk level that is less developed and less sophisticated than that of the primary insurers. Additionally, reinsurers do not have the claims handling infrastructure that primary insurers have. Assuming transaction costs are relatively low, the use of a reinsurance contract with a proper attachment point combines the primary insurers' underwriting and claims adjusting services with the reinsurers' comparative advantage at obtaining capital at low cost for very large exposures.

2. LITERATURE REVIEW

As the insuring entity becomes more and more removed from the risk that is insured, information becomes more and more costly to obtain. Fazzari et al. (1988) show the cost of capital depends on the amount of asymmetry between providers and users of capital. Information asymmetry is also used in Jean-Baptiste and Santomero (2000) when they study the case of the cost of reinsurance. They argue that information problems drive most of the risk-allocation between insurers and reinsurers. As a result, long-term relationships become optimal because they allow the inclusion of new information in the pricing of reinsurance coverage. These long-term relationships do not need to be codified exactly in a long-term contract, but can result from the renewal of short-term contracts that incorporate, at each renewal date, new information that is available about the insurable risk (distribution of frequency and severity), market conditions, changes in insurer operations, etc. (see also Boyer, 2010).

The fact that insurers have an informational advantage over their investors that is much larger than that of the reinsurers over theirs, provides a strong foundation for the primary insurers' having a higher marginal cost of capital than the reinsurers. The reason why the information asymmetry is larger for the insurers than the reinsurers comes from the optimal contract structure that we observe when there is information asymmetry. As we know, when information asymmetry exists between a policyholder and an

insurer, irrespective of whether it is in the form of moral hazard or adverse selection, it is best to have a contract that does not completely insure the policyholder. As a result, coinsurance and deductible are a rational response to information asymmetries. With reinsurance contracts, this partial insurance is even less subject to asymmetric problems since the reinsurer not only is generally assuming a portfolio of risk whereby individual idiosyncratic risk has been almost completely eliminated, but also assuming a higher tranche means that information asymmetry problems are reduced. Investors know this as well, so they should request a lower informational risk premium from reinsurance companies than from primary insurers. In our framework, the relatively small (i.e. non-catastrophe) losses require important investment in underwriting and claims adjusting expertise, but as the size of the loss grows (i.e., it approaches that of a catastrophe), underwriting and claims becomes less important and having capital becomes more important. Consequently, we can presume that individual risks do not matter as much for reinsurers; as you get to higher attachment points, capital becomes more important than underwriting expertise at the individual risk level.

Zanjani (2002) argues that capital costs are an important component of reinsurance contract pricing. Since reinsurers are potentially significantly exposed to capital outflows when a catastrophe hits, the cost of providing security for larger and larger amounts increases by more than the expected liability amount. In other words, the marginal price of insurance is an increasing function of the marginal liability.

Reinsurance contracts are often sold in layers (see Garven and Lamm-Tennant, 2003, Hurlimann 2003, and Ladoucette and Teugels, 2006). This contrasts with the approach used in Doherty and Tinic (1981) where reinsurance is examined as a bilateral risk-reducing agreement between risk-averse insurers. In a world where insurers and their providers of capital have access to capital markets, reinsurance, as a method of reducing the riskiness of returns to the owners of the insurer, becomes redundant.⁴ Insurance economists (Powell and Sommer, 2007, Berger et al., 1992, and Garven and Lamm-Tennant, 2003) have essentially seen the purchase of reinsurance as a capital structure decision, with equity capital and

⁴ The argument is similar to that of Modigliani and Miller (1957) whereby the insurers' shareholders can reduce the impact of idiosyncratic risk by diversifying their personal portfolios. If, however, risk-averse policyholders are incompletely diversified because of transaction costs or some other reason, they are willing to pay a premium that is inversely related to insurer's probability of default. Put differently, risk-averse policyholders are willing to pay a higher price for an insurance contract that originates from an insurer whose probability of default is low (Sommer, 1996).

reinsurance acting as substitutes.⁵ Given that a suboptimal capital structure (Myers and Majluf 1984) leads to undertaking value-destroying investments or foregoing value-enhancing projects, having a suboptimal amount of reinsurance should lead to a decrease in the operating efficiency of insurance entities and higher premiums for the consumers. Consequently, not only is reinsurance an important component of insurer efficiency, it can also be an important lever of public intervention.

In addition to primary insurers and reinsurers, entities that could potentially assume some catastrophic risk are the different levels of government. Because of their taxing authority, governments have the highest ability to access the capital markets and the lowest cost of bearing risk. But because of their structure, we can assume that governments have the worst underwriting ability because they are not in the business of selling insurance (see Lewis and Murdock, 1996).⁶ A similar argument can be made about insurance-linked securities (see Albertini and Barrieu, 2009, and Cummins and Weiss, 2009): ILS investors (the greater capital market participants) probably know little about the risks they are implicitly underwriting, but they have access to a lot more capital at a cheaper price. In fact, many of the insurance-linked securities have parametric triggers, modeled triggers, or dual triggers which reduce the underwriting risk in the security's payout structure and replace it with basis risk.

As public policymakers are increasingly aware of the impact of insurance availability and affordability on their constituents, government intervention in insurance markets has increased. Both the federal government and various state governments function as primary insurers (National Flood Insurance Program (NFIP), various state beach and windstorm pools) and reinsurers (Terrorism Risk Insurance Program (TRIP, TRIA), Florida Hurricane Catastrophe Fund). In addition to the insurance programs, the federal government has stepped in to provide disaster relief to areas hit the hardest by natural catastrophes. Assuming such interventions can be welfare enhancing, they must be designed so that

⁵ Traditionally, the corporate finance literature has sought to explain how corporations choose their capital structure as an optimal mix between debt and equity. Applying the same approach to insurers, insurance economists have had to adjust the financial economic model of capital structure to include a type of capital that manufacturing firms do not have: Reinsurance capital.

⁶ Political pressures may also affect a government entity providing insurance. Underwriting and claims adjusting services provided by private market insurers are usually based on actuarially sound principles. Government entities may be influenced by externalities in providing these services. Evidence of this type of political influence can be seen in the National Flood Insurance Program (see Browne and Hoyt, 2000) and Citizens Property Insurance Corporation and the Florida Hurricane Catastrophe Fund (see Cole et al, 2009b).

they become the most efficient mechanism. This is the problem modeled in this paper, a cost minimization approach of a public policymaker who seeks to structure the insurance/reinsurance market so that the total cost of purchasing insurance against a catastrophic loss is as low as possible.

3. MODEL

3.1. MODELING STRATEGY

Let us first posit that the total premium of the insurance contract includes the expected loss and any other expenses related to marketing, underwriting, claims handling, and whatever risk premium is needed to reward the providers of capital in this market. Let us also posit that the cost of the insurance contract is made up of all costs in excess of the expected loss. Put differently, the premium is given by $\Pi = E[Y] + C(Y)$ where $E[Y]$ is the expected loss and $C(Y)$ is the total cost of the insurance services. It will become obvious later why we separate this cost of insurance services (that will include underwriting and claims services as well as the implicit and explicit cost of capital requirements) from the pure premium (or the expected economic loss), which we will assume is exogenously determined and must be borne by someone in the economy. The loss Y is distributed according to some density function $g(Y)$ over the range $Y \in [0, \hat{Y}]$, where \hat{Y} is the maximum possible loss. Therefore, $E[Y] = \int_0^{\hat{Y}} Yg(Y)dY$.

We will assume in our model that all policyholders (consumers with property exposed to catastrophic risk) are trying to minimize the total cost of their insurance contract. We will assume that the total cost of insurance services, $C(Y)$, has two components: insurer expenses (the underwriting and claims administrative costs, including loss adjustment expenses) and the cost of bearing the risk. It is the relationship between these two cost components (the underwriting cost and the cost of bearing the risk) that determines the insurance/reinsurance contract structure.

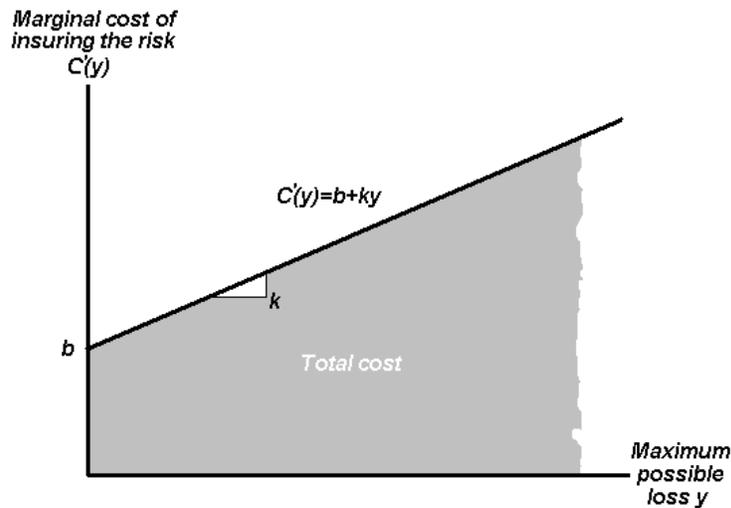
The model assumes there are N potential entities that could sell insurance protection in a competitive market, where the marginal price of insurance is equal to its marginal cost (see Zanjani, 2002). For

simplicity, assume that each of these entities is characterized by a linear marginal cost⁷ of providing coverage in the event of a catastrophic loss. This marginal cost function depends on the insurer's cost of capital (which we shall denote k) and its underwriting and claims-handling ability (which we shall denote b). For any of these entities, n , the **marginal cost** associated with a possible loss of magnitude Y is a linear function with two parameters, given by Equation 1 and illustrated in Figure 3.1.

$$C_n^l(Y) = b_n + k_n Y \quad (1)$$

with $Y \in [0, \hat{Y}]$ where \hat{Y} is the maximum possible loss. Entities in the economy differ with respect to their b_n 's and k_n 's as determined by the entity's production function (discussed later). The goal of the policyholder is to minimize total cost of insuring against a possible loss \hat{Y} .

Figure 3.1: Linear Marginal Cost Function



In other words, the policyholder chooses an insurance contract, or a set of insurance contracts, that minimizes the integral of the marginal cost function. The approach we use can be seen as a simplified

⁷ The marginal cost does not need to be a linear function of the maximum possible loss; the results will hold as long as the marginal cost remains an increasing function of the maximum possible loss.

version of the pricing model developed by Zanjani (2002).⁸ The total cost of bearing some maximum possible loss \hat{Y} is the area under the marginal cost function plus a constant term, which we can represent as the fixed cost of organizing an insurance system that insures individuals in society (rent, overhead, etc).

3.1.1 Single Insurance Provider

If there is only one type of entity in the economy that can sell insurance (with $b_n = b$ and $k_n = k$) and assuming a first-dollar insurance contract (that is, the first dollar of loss is assumed by the insurer), society's problem is then simply to minimize Equation 2.

$$\int_0^{\hat{Y}} [b + kY] dY \quad (2)$$

The premium charged by the single provider would then be

$$\Pi = E[Y] + C(Y) = \int_0^{\hat{Y}} Yg(Y)dY + \int_0^{\hat{Y}} [b + kY]dY = \int_0^{\hat{Y}} [Yg(Y) + b + kY]dY$$

In the case of a more general marginal cost function, say a convex function given by $C'(Y) = b + f(Y)$,

with $f'(Y) > 0$ and $f''(Y) > 0$, the total premium would be equal to $\Pi = \int_0^{\hat{Y}} [Yg(Y) + b + f(Y)]dY$.

3.1.2 Two Insurance Providers

Now suppose there are two entities n_1 and n_2 such that $b_1 < b_2$ and $k_1 > k_2$. This means that entity n_1 's marginal cost intercept is lower than entity n_2 's. Put differently, entity n_1 is able to provide underwriting and claims service marginally cheaper than entity n_2 . However, each dollar of coverage (marginal cost of capital) is more expensive for entity n_1 . The question becomes how to combine the two entities' technology to minimize the total cost of the risk. Because one entity has a lower intercept but a higher slope, a policyholder will minimize the total cost by dealing with the low-intercept entity (better

⁸ See Appendix A.

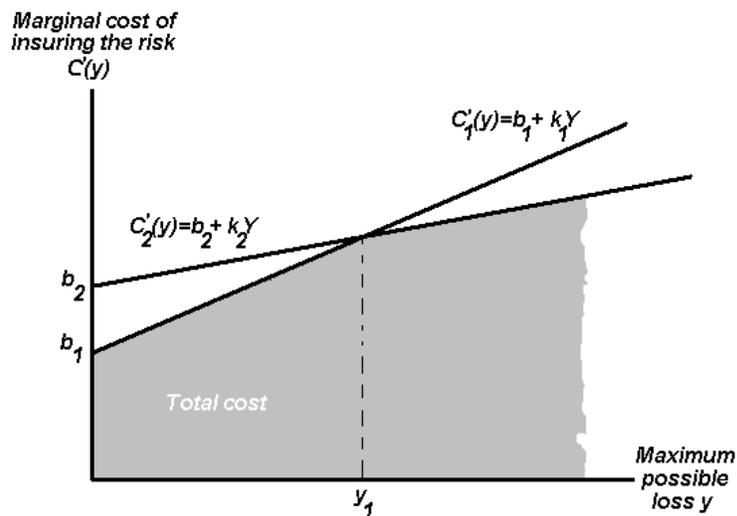
underwriting and claims service) for lower losses and the low-slope entity (lower marginal cost of capital) for higher losses.

Graphically, we find that the total cost of bearing risk of potential loss \hat{Y} is a combination of the two entities: The low-intercept entity is responsible for losses up until point y_1 and the low-slope entity is responsible after point y_1 . Changing vocabulary to fit with the insurance industry's we can say that entity n_1 is the primary insurer whereas entity n_2 is the reinsurer that assumes losses greater than y_1 . The question becomes: At what attachment point should the reinsurer become liable? Put differently, where should y_1 be to minimize the total cost of bearing this risk? Abstracting from the expected loss component of the total premium, which we assume to be exogenously given, social welfare is maximized by minimizing the total cost of providing insurance to policyholders. The minimization problem for society then becomes Equation 3,

$$\text{Min}_{y_1} \int_0^{y_1} [b_1 + k_1 Y] dY + \int_{y_1}^{\hat{Y}} [b_2 + k_2 Y] dY \quad (3)$$

and is shown graphically in Figure 3.2.

Figure 3.2: Two insurance providers



The total premium that policyholders would pay in the case of one primary insurer responsible for the losses up to y_1^* , which has been chosen optimally to minimize the total cost (i.e.,

$y_1^* \in \arg \min_{y_1} \int_0^{y_1} [b_1 + k_1 Y] dY + \int_{y_1}^{\hat{Y}} [b_2 + k_2 Y] dY$), and of one reinsurer responsible for the losses between

y_1 and \hat{Y} is given by $\Pi = E[Y] + C(Y) = \int_0^{\hat{Y}} Y g(Y) dY + \int_0^{y_1^*} [b_1 + k_1 Y] dY + \int_{y_1^*}^{\hat{Y}} [b_2 + k_2 Y] dY$. The premium

collected by the reinsurer is $\Pi_R = \int_{y_1^*}^{\hat{Y}} Y g(Y) dY + \int_{y_1^*}^{\hat{Y}} [b_2 + k_2 Y] dY$. The amount saved (Π_S) by having losses

greater than y_1^* reinsured instead of having all the risk being borne by the primary insurer is given by

$\Pi_S = \left(\int_0^{\hat{Y}} Y g(Y) dY + \int_0^{y_1^*} [b_1 + k_1 Y] dY \right) - \left(\int_0^{\hat{Y}} Y g(Y) dY + \int_0^{y_1^*} [b_1 + k_1 Y] dY + \int_{y_1^*}^{\hat{Y}} [b_2 + k_2 Y] dY \right)$. More concisely

savings are equal to $\Pi_S = \int_{y_1^*}^{\hat{Y}} [b_1 + k_1 Y] dY - \int_{y_1^*}^{\hat{Y}} [b_2 + k_2 Y] dY = \int_{y_1^*}^{\hat{Y}} [(b_1 - b_2) + (k_1 - k_2) Y] dY$.

Note that if the property owner chooses to retain the first portion of the risk (a deductible) and then insure above that point, the function to minimize would then simply be equation 4,

$$\text{Min}_d \int_0^d [b_0 + k_0 y] dy + \int_d^{\hat{Y}} [b_1 + k_1 y] dy \quad (4)$$

with d being the deductible and replacing y_1 . We can therefore see this case as that of a policyholder who becomes the first-dollar insurer who then reinsures the risk with what we have called the primary insurer. The total premium paid by the policyholder would then be given by what we called previously

the reinsurer's premium, $\Pi_R = \int_d^{\hat{Y}} Y g(Y) dY + \int_d^{\hat{Y}} [b_2 + k_2 Y] dY$, where the attachment point y_1 is replaced

by the deductible d . The implicit total premium paid by the policyholder is not only that paid to the insurer, but also includes the portion that the policyholder retains. Consequently, the total premium-

cum-cost for society does not change and remains $\Pi = \int_0^{\hat{Y}} Y g(Y) dY + \int_0^d [b_0 + k_0 Y] dY + \int_d^{\hat{Y}} [b_1 + k_1 Y] dY$,

where the index 0 represents the policyholder's marginal cost function of bearing risk.

It is interesting to note that we have a new reason why deductible exists in a competitive environment. Even with no adverse selection or moral hazard problems, because insurers have lower capital cost of bearing risk than individuals (who are better equipped to assess their own risk), individuals will assume the first few dollars of loss whereas insurers will step in as the providers of resources when losses are greater than the threshold d found in equation 4.⁹

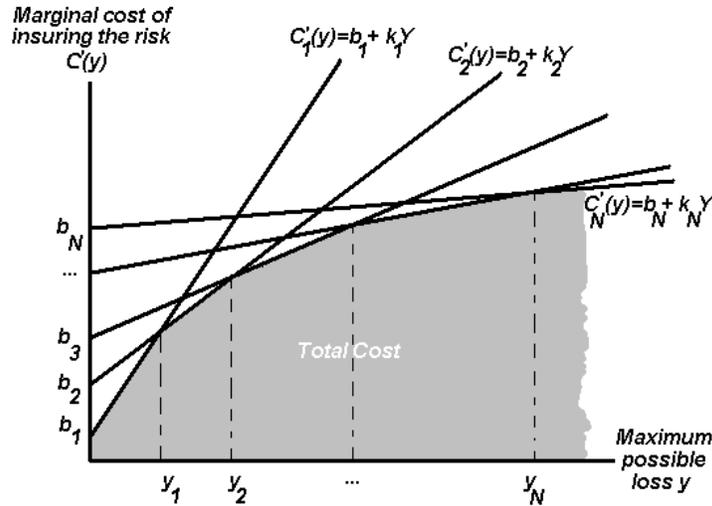
3.1.3 N Insurance Providers

Now suppose there are N entities such that $b_1 < b_2 < \dots < b_N$ and $k_1 > k_2 > \dots > k_N$. This means that entity n_1 's marginal cost intercept is lower than entity n_2 's, which is lower than n_3 's, etc. Each dollar of coverage (marginal cost of capital) is more expensive for entity n_1 than for n_2 than for n_3 etc. As before, the optimal combination of the N entities' technology will be for the policyholder to deal with the entity that has the lowest intercept first (it has the best underwriting and claims service technology), and then as reinsurance steps in at different layers when having a low marginal cost of capital becomes important. Layers are determined by the comparative advantage of each reinsurer at assuming catastrophic losses as shown in Figure 3.3.

Reinsurance in this economy "concavifies" the overall marginal cost function. By increasing the number of entities (i.e. reinsurers) one increases the concavity of the marginal cost function and therefore reduces total cost. The resulting curve in figure 3.3 could be thought of as a contract efficiency (efficient-C curve) curve. This curve could be used to compare actual insurance programs to this minimum cost curve. The insurance contracts would lie up and to the left of this curve and a measure of the efficiency loss would be the difference in the areas under the two curves. It would be possible to determine at which points in the loss distribution inefficiencies are created and whether those inefficiencies are due to capital (k) or labor (b) issues, or inefficient attachment points.

⁹ This still assumes some information asymmetry, but no residual asymmetry after underwriting expertise. If there was no costly information, underwriting would not be costly, but claims handling would be, still including some level of inefficiency. For very small losses, individuals can use their checking account (or line of credit or credit card) to handle small claims better than insurers.

Figure 3.3: N insurance entities



As the market allows more and more insurers that have different underwriting expertise (b) and risk bearing capacities (k) the total cost to policyholders, still excluding the pure premium, is decreased. This necessarily improves everyone's welfare. If there are N private insurers and reinsurers such that $b_1 < b_2 < \dots < b_N$ and $k_1 > k_2 > \dots > k_N$, society's cost minimization problem is equation 5.

$$\text{Min}_{y_1, \dots, y_N} \left\{ \int_0^{y_1} [b_1 + k_1 Y] dY + \sum_{i=2}^N \int_{y_{i-1}}^{y_i} [b_i + k_i Y] dY \right\} \quad (5)$$

3.2 AN EXOGENOUS EQUILIBRIUM

Clearly the equilibrium on this market will depend on how the parameter values b_i and k_i of all private insurers and reinsurers are distributed in the economy. Suppose insurer h , with b_h and k_h , and insurer j with b_j and k_j . If $b_h < b_j$ and $k_h < k_j$, then cost minimization will be obtained by having only one insurer. In other words, insurer h here dominates insurer j for whatever type of loss: It has better underwriting expertise and a lower cost of bearing risk. In an efficient market, insurer j would find itself filing for bankruptcy.

Suppose now that $b_h < b_j$ and $k_h = k_j$ so that both insurers have the same risk bearing technology, but one insurer (insurer h) has a better underwriting expertise than the other. In other words, one insurer can do the same underwriting job, but at a lower cost. Again, insurer j would find itself filing for bankruptcy since it has a more costly production function than insurer h . A similar story can be told if $b_h = b_j$ and $k_h < k_j$, so that both insurers have the same underwriting ability, but one insurer (insurer h) has a better ability to assume large losses than the other insurer in the sense that insurer h 's cost of assuming the risk is lower. Clearly insurer j would find itself filing for bankruptcy, again, since it has a more costly production function than insurer h .

For a reinsurance market to exist in equilibrium, it therefore has to be that the reinsurers' marginal cost functions have a higher intercept and a lower slope. If this is not the case, then the entire potential loss of a policyholder will be assumed by a single unique insurer. In reality, we know that primary insurers rely on reinsurers to guarantee eventual indemnity payments for the highest levels of potential losses. Consequently, in the absence of market imperfections a policyholder's loss will be handled by more than one entity only if reinsurers have a lower cost of bearing large risks than primary insurers.

Assume now that the two insurers have the same b and the same k . If two insurers have the same marginal cost function, this means that there is no value in excess of loss reinsurance since there is nothing to gain. The primary insurance market would then, on average, be split between the two insurers who are both offering the insurance service at the lowest possible marginal cost to the policyholders. Imagine that there is a third entity in this market that has a lower b and a higher k than these two. If that is the case, then the new entity would become the primary insurer (having the lowest intercept) and the two others would become reinsurers that each receives half of the primary insurer's business. One can imagine that this fits the description, from the point of view of the reinsurer, of a proportional reinsurance contract with each reinsurer assuming 50% of the loss above the attachment point (which is sometimes referred to as corridor contracts). If instead of having a lower intercept the third entity has a lower marginal cost slope (and a higher intercept) than the first two insurers, then the primary market would be split equally between the two initial insurers and both would reinsure their higher losses with the new entity using an excess of loss contract.

By adding further and further insurance entities that have different b 's and k 's gives us a market equilibrium whereby primary insurers are those that have the lowest b 's and where reinsurance companies' involvement through excess-of-loss contracts depends on the right combination of b 's and k 's, with the reinsurer with the lowest k and the highest b assuming the highest tranche. If two or more entities have the same b and the same k , then they split equally the tranche in which they belong in the marginal cost hierarchy. Appendix B shows this relationship in an insurance program chart.

If there are market imperfections, such as search costs, then it is quite possible that the optimal structure that minimizes marginal cost is not obtained. It nonetheless remains theoretically feasible to find the combination of insurers and reinsurers that minimizes the total cost of supporting catastrophic risk. Whether this optimal combination is observed in reality or not becomes an empirical question that can be answered in a companion paper.

3.3 AN ENDOGENOUS EQUILIBRIUM

3.3.1 An Insurance/Reinsurance Production Function

A question at this juncture is how do firms decide to offer insurance service as a primary insurer or as a reinsurer, and as what type of reinsurer (high attachment point or low attachment point insurer). In the previous section we just assumed that some entities had high b 's and low k 's (the reinsurers typically) whereas others had low b 's and high k 's (the primary insurers typically).

Imagine that the genesis of the insurance market is populated by a set of entities that all have access to the same technology that is given by some function $T_n(K, L) = K^{\alpha_n} L^{1-\alpha_n}$ for $n \in \{1, \dots, N\}$. For simplicity let us assume a constant elasticity of substitution production function. All entities want to maximize their value by choosing the right amount of capital K_n and labor L_n .¹⁰ Entities differ only with respect to the parameter $\alpha_n \in (0, 1)$. Assume that all entities start with the same level of surplus, S

¹⁰ As it will become apparent later, we should view labor as the investment an insurer makes in the underwriting and the claims handling abilities of its employees whereas capital should be viewed as the energy it puts in optimizing its capital structure, ability to pool individual risks, diversify risk by line of business and geographically, and attract capital to meet the current level of risk it seeks to assume.

(which we could also see as their available capacity). The price of capital is given by p_k , which we will assume constant for whatever level of capital, and the price of labor is given by p_L , which will always be constant per unit of labor. Consequently, an entity n will choose a level of labor and capital that at most uses the entire insurer's available surplus so that $p_k K_n^* + p_L L_n^* \leq S$. As entities are endowed with technology that allows them to be more or less efficient in the use of labor or capital (the parameter α varies from one entity to the next), they will opt to invest more in one and less in the other. A firm's problem can then be written as a choice between investment in capital and investment in labor that maximizes firm value:

$$\max_{K_n, L_n} T_n(K, L) = K_n^{\alpha_n} L_n^{1-\alpha_n} \quad s.t. \quad p_k K_n + p_L L_n \leq S. \quad (6)$$

The solution to this problem is straightforward.¹¹ Firm value is maximized when the amount invested in labor and in capital is such that

$$L_n^* = \frac{(1-\alpha_n)}{P_L} S \quad \text{and} \quad K_n^* = \frac{\alpha_n}{P_K} S.$$

We see that as the parameter α_n becomes larger, an entity will invest more in capital. At the other end of the spectrum, a low parameter α_n means that the entity has a better underwriting technology and therefore will invest more in the labor component.

An alternative modeling approach would be to see S as the amount of economic capital needed to support a given risk. In a CAPM world, we know that for a given risk, the amount of economic capital needed is independent of the insurer since it depends only on the covariance of the risk with the market portfolio. The same is true in a reinsurance context as shown in Borch (1962). This alternative approach allows for the modeling of each risk individually so that (re)insurance entities are allowed to assume

¹¹ The first order conditions write $\alpha_n K_n^{\alpha_n-1} L_n^{1-\alpha_n} - \lambda p_k = 0$ and $(1-\alpha_n) K_n^{\alpha_n} L_n^{-\alpha_n} - \lambda p_L = 0$. Solving we find a Lagrange multiplier equal to $\lambda = (1-\alpha_n) \left(\frac{K_n}{L_n} \right)^{\alpha_n} \frac{1}{p_L} = \alpha_n \left(\frac{K_n}{L_n} \right)^{\alpha_n-1} \frac{1}{p_K}$. This yields an optimal choice of capital and labor such that $K_n = \frac{\alpha_n}{1-\alpha_n} L_n \frac{p_L}{p_K}$ and $L_n = \frac{1}{p_L} S - \frac{p_k}{p_L} K_n$. Solving for K_n and L_n completes the exercise.

different layers for different risks. To see why, we could let the parameter $\alpha_{n,m} \in (0,1)$ be different for each risk m that requires surplus S_m to underwrite. Since the choice of K and L is made for each risk individually as a function of the surplus that is required and the (re)insurer's production function, (re)insurers could make different K and L choices as a function of the type of risk.

3.3.2 Returning to the Marginal Cost Function

Letting $b_n = \frac{1}{L_n^*}$ and $k_n = \frac{1}{K_n^*}$ ¹² in the marginal cost equation we had before, we see that an entity that is endowed with a higher α_n parameter will have a marginal cost function that has higher intercept and a lower slope, the type of cost function that one should observe in a reinsurer. The opposite also fits our model as firms whose parameter α_n is small will be more likely to become primary insurers since their marginal cost function will have a lower intercept and a higher slope.

An interesting aspect of this insurance production function is that we can see that a sudden increase in the unit price of capital (p_k) will reduce the amount of capital that every entity uses, but it will not affect the amount of labor used. This means that as we transpose the production function into the cost function that society wants to minimize, a capital shock does not alter the intercept of the marginal cost function, but it does increase its slope, consistent with an increase in the capital cost of bearing large risks. This impact will be larger for companies that already invest a lot in the capital component of the production function (that is, the reinsurers). To see why, note that $\frac{\partial K_n^*}{\partial p_K} = -\frac{\alpha_n}{(p_K)^2} S < 0$. As we know

from an earlier discussion, entities that have a large α_n parameter are those that invest more in capital, and that are more likely to be reinsurers. It then follows that large α_n entities (i.e., the reinsurers) will be more affected by capital price shocks than primary insurers, which is consistent with industry stylized facts as well as the literature on reinsurance capital (see Berger et al., 1992).

¹² This assumes that investing more in labor (capital) lowers the marginal cost of providing labor (capital) to the policyholders.

Following our particular setup, we can write that $C_n^l(Y)$ is a function of labor and capital so that

$$C_n^l(Y) = b_n + k_n Y = \frac{P_L}{(1-\alpha_n)S} + \frac{P_K}{\alpha_n S} Y.$$

The comparative static shows that an increase in the price of labor increases the intercept whereas an increase in the price of capital increases the slope. Interestingly as well, an increase in the “capital intensity” parameter (α_n) gives us a higher intercept and a lower slope. Finally, larger firms, as measured by their surplus, should have a lower intercept and a lower slope, suggesting that larger firms are better both at the underwriting end of the business and at the risk bearing end. In other words, there is an economy of scale.

The advantage of the production function we have used is that it turns a two-parameter (b_n and k_n) firm-specific marginal cost problem into a one-parameter firm-specific problem (α_n) without altering the desired properties of the distribution of the marginal cost functions. In other words, if we were to rank the firms according to the parameter α_n so that $\alpha_1 < \alpha_2 < \dots < \alpha_N$, we would have that $b_1 < b_2 < \dots < b_N$ and $k_1 > k_2 > \dots > k_N$. This means that the greater the “capital intensity” parameter (α_n), the higher is the intercept and the lower the slope. Consequently, firms that have a higher ability to use capital (i.e., firms that have a higher α_n), should become reinsurers.

A second important advantage of the production function we have chosen is that surplus is additive over the different lines of business. In other words, assuming there are M lines, firm n 's total surplus is given by $S_n = \sum_m S_{n,m}$.¹³ Allowing firms to have parameters $\alpha_{n,m}$ that differs across lines, the total amount

spent in labor (resp. capital) by the firm would then be $L_n = \sum_m L_{n,m} = \sum_m \frac{(1-\alpha_{n,m})}{P_{L,m}} S_{n,m}$ (resp.

$K_n = \sum_m K_{n,m} = \sum_m \frac{\alpha_{n,m}}{P_{K,m}} S_{n,m}$) where we assumed that labor costs (resp. capital costs) are line specific.

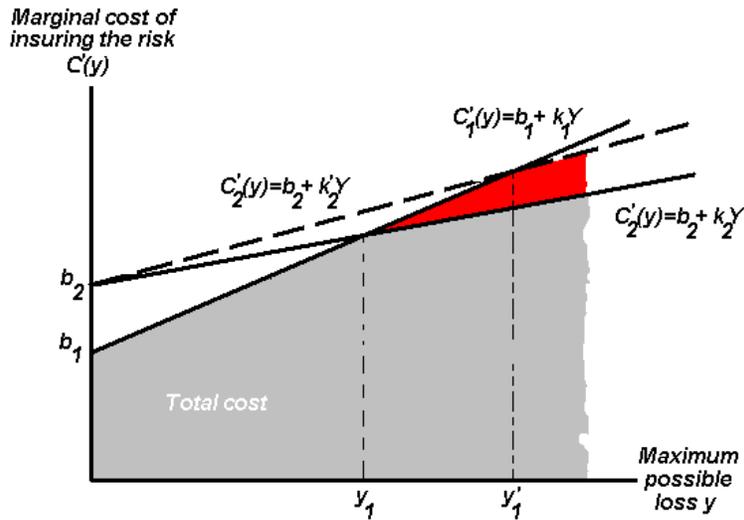
3.3.3 Some Comparative Statics

An increase in the price of capital that reduces investment in capital will translate into a higher slope of the marginal cost function. The impact will be larger for firms that have a small slope since it is for them

¹³ This is in the spirit of economic capital allocation whereby one would like to allocate capital to lines of business according to their marginal impact.

that the shock will feel worse. The impact of an increase in the price of capital that increases the slope of the marginal cost function of all insurers will not only result in a higher cost of insurance (independent of the expected loss), but also result in a higher attachment point. Figure 3.5 illustrates the situation where the primary insurer's marginal cost function was not affected, but the reinsurer's marginal cost function sees its slope increase from k to k' .

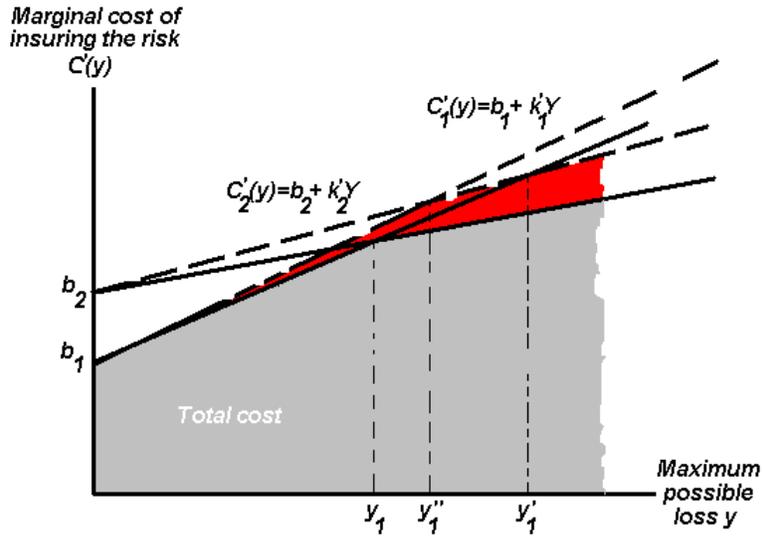
Figure 3.5: An increase in the reinsurer's cost of capital increases retention



As we can see, the reinsurance contract's attachment point increases to y_1' . The increase in the total cost of insurance is illustrated by the red area. In this graph, the slope of the primary insurer's marginal cost function was not altered. If it was altered, the total cost would increase even more, but the attachment point would not increase as much. Figure 3.6 illustrates the case where both the primary insurer's as well as the reinsurer's marginal cost slope increase. The region in red is the region that represents the extra cost to policyholders.

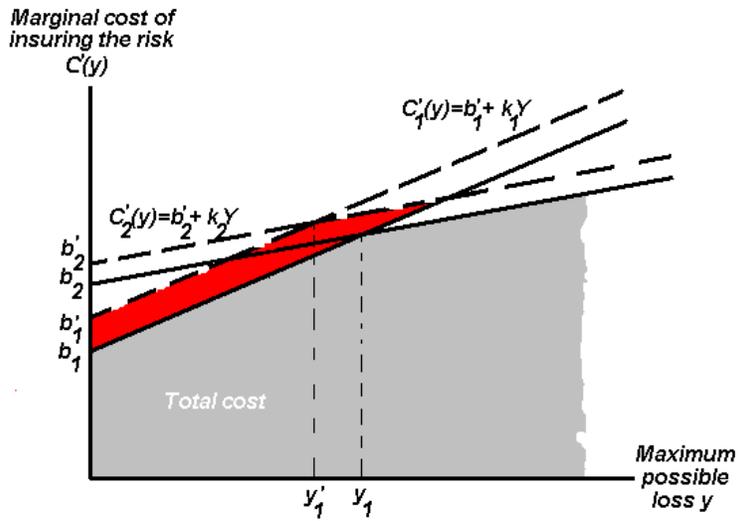
It is important to remember that the increase in the cost has nothing to do with an increase in the size of the loss or a different loss distribution. The increase here is only associated with an increase in the price of capital and therefore only affects the cost of providing insurance independent of the pure premium or the expected loss.

Figure 3.6: An increase in the primary insurer's and the reinsurer's cost of capital increases retention



If the market was to experience a higher cost of labor, the intercept would increase but the slope would not change. The increase would be larger for the primary insurer than for the reinsurer since the primary insurer's production function is more labor intensive. An increase in the unit cost of labor would increase the total cost of insurance, but reduce the attachment point as we see in Figure 3.7.

Figure 3.7: An increase in the primary insurer's and the reinsurer's underwriting cost reduces retention



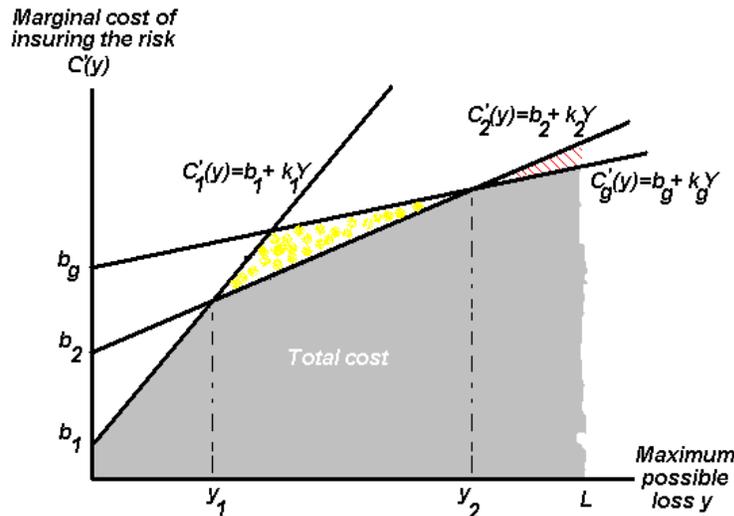
4. AN APPLICATION TO PUBLIC INTERVENTION

4.1 The Role of Government as an Insurance Provider

In this model, government entities can enter as insurance entities. We are assuming that a government entity has the lowest cost of raising capital (so it has the lowest marginal cost of bearing risk, k_g) but it has the highest underwriting cost since it has no expertise in the matter (so it has the highest intercept, b_g). Therefore, it would enter as the reinsurer of last resort. Expanding the two-provider model with the third entity being a government insurance provider, we learn that the problem for society is to find the reinsurer's appropriate attachment point y_1 and detachment point y_2 such that we still minimize the total cost as shown in equation 7 and graphically in Figure 4.1.

$$\text{Min}_{y_1, y_2} \int_0^{y_1} [b_1 + k_1 y] dy + \int_{y_1}^{y_2} [b_2 + k_2 y] dy + \int_{y_2}^{\hat{Y}} [b_g + k_g y] dy. \quad (7)$$

Figure 4.1: Government Entity as Insurer



If reinsurance is not allowed, but government is still there as a reinsurer of last resort, the total cost would be higher by an amount that is represented in the graph by the yellow triangle. The government's

marginal contribution to the reduction in total cost can also be measured as the lined area in red on the graph. Without the government as a reinsurer of last resort, private reinsurers would have to assume the risk from attachment point y_1 until the maximum possible loss \hat{Y} . Thus, the total cost to insuring the loss would be greater by an amount that is represented by the lined red triangle.¹⁴

If there are N private insurers and reinsurers such that $b_1 < b_2 < \dots < b_N$ and $k_1 > k_2 > \dots > k_N$, and a government, whose parameters are b_g and k_g such that $b_N < b_g$ and $k_N > k_g$, that acts as a reinsurer of last resort, society's cost minimization problem becomes equation 8.

$$\text{Min}_{y_1, \dots, y_N} \left\{ \int_0^{y_1} [b_1 + k_1 y] dy + \sum_{i=2}^N \int_{y_{i-1}}^{y_i} [b_i + k_i y] dy + \int_{y_N}^{\hat{Y}} [b_g + k_g y] dy \right\} \quad (8)$$

4.2 PUBLIC POLICY IMPLICATIONS

The question in terms of public policy will be to assess the parameter values b_i and k_i of all private insurers and reinsurers, as well as the government's, so that the government's optimal attachment point can be determined. With this type of model, where competition in the primary layer and working layers of reinsurance are dominated by firms with better underwriting and claims adjusting capabilities, there are no advantages to having a government entity provide insurance coverage. It is also possible that there is no point for government to become involved in the insurance market as a reinsurance of last resort if, for instance, we find that cost minimization is obtained in the private market because the solution would demand that $y_N = \hat{Y}$. However, as the maximum possible loss increases, it becomes more likely that a government entity is needed in the market as its lower cost of capital begins to outweigh its inability to underwrite and manage claims.

The public policy implications of the impact of having different levels of government involved in the supply of insurance capital are not trivial, even if one abstracts from the moral hazard and adverse

¹⁴ The results presented above hold even if the marginal cost is not a linear function of the maximum possible loss, provided that the marginal cost remains an increasing function of the maximum possible loss. As the market allows more and more insurers that have different underwriting expertise (b) and risk bearing capacities (k), the total cost to policyholders, excluding the pure premium, is decreased.

selection problems. Public intervention will have an impact on the price of insurance and on the wellbeing of insurers, reinsurers, and policyholders. It will also have an impact on the tax base as every individual in the state or in the country becomes an “investor” of the government-as-(re)insurer. With the discussions of multi-state catastrophe pools or a federal catastrophe pool, the roles of insurers, reinsurers and public entities increasingly becomes a public policy issue. A more exhaustive study of the optimality of attachment and detachment points can aid public policymakers in making decisions in the best interests of their constituents.

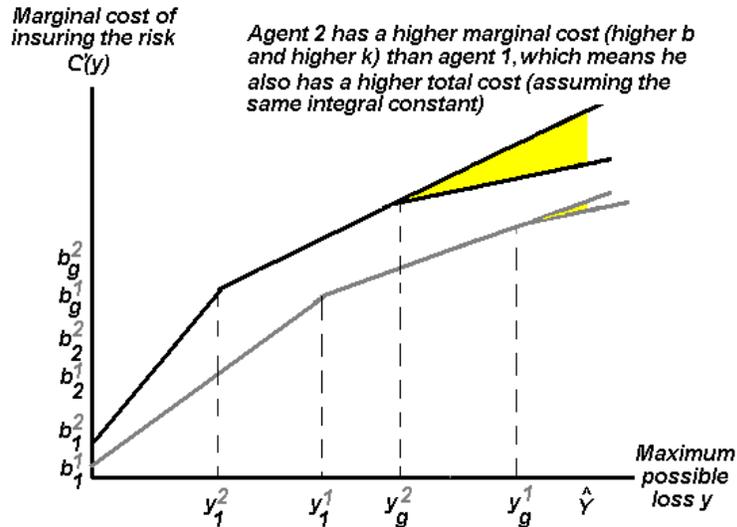
The question of government intervention cannot be studied independently of the distribution of risk in the economy. In the model so far, all individuals face the same risk, which means that government intervention has no ex-ante redistribution impact. As a result, provided that at some level the government’s cost of capital is lower than the reinsurers’ lowest, government intervention increases welfare. Suppose now that agents in the economy are heterogeneous with respect to the cost of providing them with insurance. Put differently, suppose that there is a proportion μ_θ of agents (with $\sum_\theta \mu_\theta = 1$) whose total cost of insurance services is given by $C^\theta(Y)$. All agents still face the same expected loss, but some are more costly to insure. Using the case of one primary insurer, one reinsurer and government (who cost of capital is independent of the private market’s cost function), the problem to minimize becomes

$$\text{Min}_{y_1^\theta, y_g^\theta} \int_0^{y_1^\theta} [b_1^\theta + k_1^\theta y] dy + \int_{y_1^\theta}^{y_g^\theta} [b_2^\theta + k_2^\theta y] dy + \int_{y_g^\theta}^{\hat{Y}} [b_g^\theta + k_g y] dy,$$

where the superscript represent the agents’ “cost type”, and the subscript g refers to the situation facing the government.

The optimal contract that minimizes the total cost of insurance will differ from one agent type to the next as the attachment and detachment points will not be the same for every contract, which is represented in the minimization function by the superscript on the attachment and detachment points. If the government was able to offer different protection (different attachment points) to different agent-types, the allocation of total cost in the economy would be Pareto optimal as each agent would end up paying a total cost that is specific to him (as shown in Figure 4.2).

Figure 4.2: Gain from government intervention



In reality, however, governments rarely treat different agents differently. Instead, governments usually use a one-size-fits-all approach in its policies (which may reflect its inability to (underwrite?) discriminate properly across types).

Since most government run property casualty insurance programs involve some subsidization of high risk exposures, there are also redistributive questions that need to be addressed. In Florida (see Maroney, Nyce and Schneider, 2009), inland homeowners subsidize homeowners who live on the coast, and even properties slightly inland in the coastal are subsidizing properties that are directly on the ocean.

There are two types of government involvement that would induce redistribution problems. In the first intervention, we will assume that government intervenes at the same level of loss for all agent types (that is, the government's attachment point is the same for all). In the second, we will assume that government charges the same marginal cost to all the agents (that is, the parameters b_g and k_g are the same for all agents).

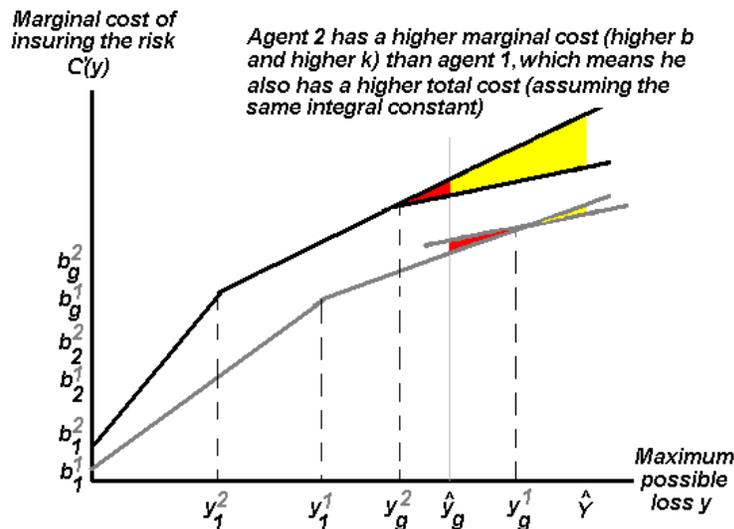
4.2.1 Same government protection (i.e., same attachment point)

In the model, the government's inability to discriminate results in every agent facing a government attachment point of \hat{y}_g determined exogenously. Each agent-type's problem can then be written as

$$\text{Min} \int_{y_1^\theta}^{y_1^\theta} [b_1^\theta + k_1^\theta y] dy + \int_{y_1^\theta}^{\hat{y}_g} [b_2^\theta + k_2^\theta y] dy + \int_{\hat{y}_g}^{\hat{y}} [b_g^\theta + k_g y] dy .$$

As we see, government intervention fixes the upper attachment point \hat{y}_g so that it is no longer a choice variable in the problem. If government fixes its attachment point \hat{y}_g between the optimal attachment points of each type of agent, it is then easy to show that every agent ends up paying more for insurance services. To see why, observe the following Figure 4.3. The red wedges represent the extra cost imposed on each agent by having a fixed attachment point and the yellow represents the gain to each agent for having government intervention.

Figure 4.3: Gain and loss from government intervention

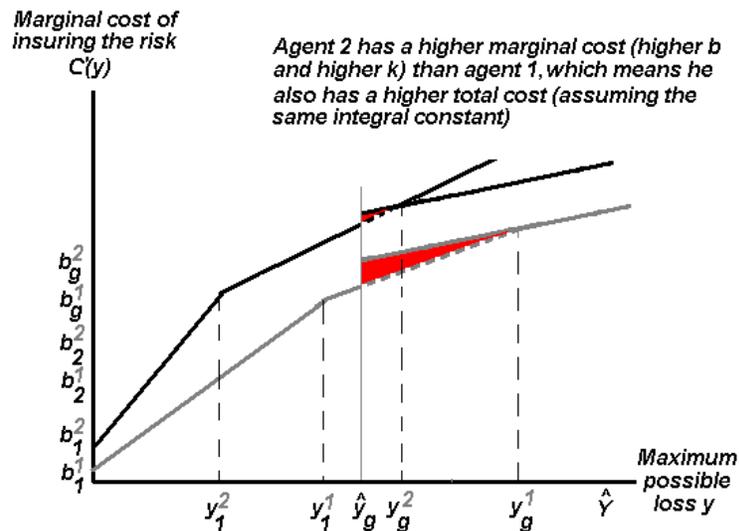


As we see, the government's attachment point \hat{y}_g lies between the two type specific (and optimal) attachment points y_g^2 and y_g^1 . This means that, compared to the optimal type-specific entry point, government intervenes too early for the agents that have the lowest marginal cost (agent-type 1) and too late for the agents that have the higher marginal cost function (agent-type 2). As a result, both types

of agents end up with a suboptimal situation. The loss of welfare for society is then given by the sum of the two areas highlighted in red.

Interestingly, no government intervention that fixes its entry point can be Pareto optimal. To see why, suppose that $\hat{y}_g < y_g^2$. The situation would then look like that of Figure 4.4.

Figure 4.4: Government Intervention by fixing its attachment point below y_g^2



As we can see, neither agent benefits from the government stepping in too early in the catastrophe risk market. In fact, whatever entry point government fixes, agents can never be better off if that entry point is the same for all.

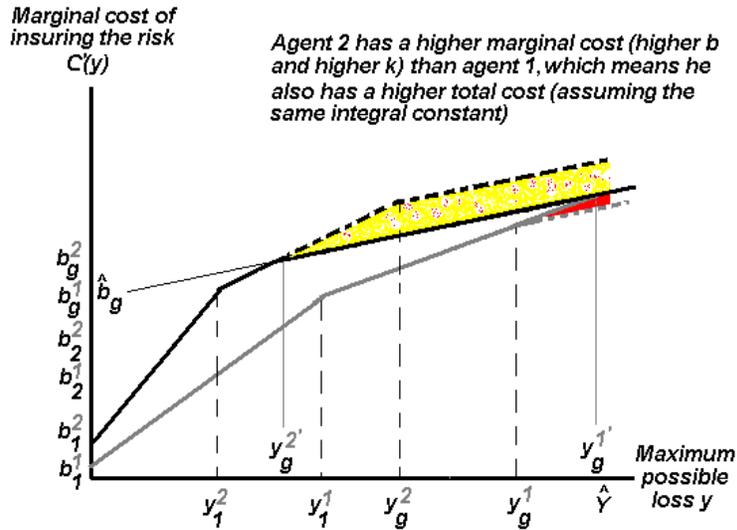
4.2.2 Same marginal cost of government insurance

The second type of redistribution the government can do is to forgo its ability to charge agents as a function of their marginal cost type. Instead the government may use an “average” cost for all. Given the way we have modeled the problem here, this means that the government inability to discriminate results in every agent facing a government average “underwriting expertise cost” of $b_g = \sum_{\theta} \mu_{\theta} b_g^{\theta}$. The problem for each agent-type then becomes

$$\text{Min}_{y_1^\theta, y_2^\theta} \int_0^{y_1^\theta} [b_1^\theta + k_1^\theta y] dy + \int_{y_1^\theta}^{y_2^\theta} [b_2^\theta + k_2^\theta y] dy + \int_{y_2^\theta}^{\hat{Y}} [b_{\hat{g}} + k_g y] dy$$

As shown in Figure 4.5.

Figure 4.5: Government Intervention by assigning the same underwriting cost to all



By using the same intercept for all agents, the government's attachment point for the high cost agents (agent-type 2) decreases, but is increases for the low cost agents (agent-type 1). By doing so the high-cost agents are benefiting from the intervention, to the detriment of the low cost agent. Each high-cost agent's decrease in total cost is given by the area in yellow. Each low-cost agent's increase in total cost is given by the area in red. The question, from society's point of view, is whether the weighted sum of the area in yellow and the area in red is positive (weighted by the proportion of each type of agents in society), μ_θ .¹⁵

¹⁵ We can combine the two types of intervention (same attachment point, same marginal cost function) and examine how that affects the agents' choice of insurance contracts. The problem then becomes

$$\text{Min}_{y_1^\theta} \int_0^{y_1^\theta} [b_1^\theta + k_1^\theta y] dy + \int_{y_1^\theta}^{\hat{y}_2} [b_2^\theta + k_2^\theta y] dy + \int_{\hat{y}_2}^{\hat{Y}} [b_{\hat{g}} + k_g y] dy, \text{ where } b_{\hat{g}} \text{ is defined as before as } b_{\hat{g}} = \sum_{\theta} \mu_{\theta} b_g^{\theta}.$$

5. CONCLUSION

Financing of catastrophic risk is increasingly becoming a public policy issue at the state and federal level. The growth of residual markets in hazard prone areas increases the importance of finding the proper role and price for private market insurance. This paper addressed the following four questions regarding the market for catastrophe insurance. 1- What do insurers bring to the table if it is not capital and underwriting expertise? 2- Where are the optimal attachment and detachment points for reinsurance? 3- When should reinsurance be layered and when should it be proportional? 4- Should the different levels of government be involved in catastrophic risk financing and if so, how and at what level?

This paper presented a theoretical model of the minimum cost of providing catastrophic insurance coverage through the primary and the reinsurance market that includes an implicit (or explicit) presence of governments as reinsurers of last resort. Using labor (underwriting and claims adjusting costs) and capital (risk financing) as the main inputs for providing insurance services, we showed how reinsurance is optimally layered (with attachment and detachment points) for a given book of business. Even though attachment and detachment points are determined to minimize the cost of insurance protection, the cost of catastrophic insurance can nevertheless be extraordinarily high so that making the implicit government's guarantee explicit can reduce this cost and increases the policyholders' (and thus society's) welfare.

The public policy implications of the impact of having different levels of government involved in the supply of insurance capital are not trivial, even if one abstracts from the moral hazard and adverse selection problems. Public intervention will have an impact on the price of insurance and on the wellbeing of insurers, reinsurers, and policyholders. It will also have an impact on the tax base as every individual in the state or in the country becomes an "investor" of the government-as-(re)insurer. With the discussions of multi-state catastrophe pools or a federal catastrophe pool, the roles of insurers, reinsurers and public entities increasingly becomes a public policy issue. A more exhaustive study of the optimality of attachment and detachment points can aid public policymakers in making decisions in the best interests of their constituents.

6. APPENDICES

6.1. APPENDIX A: A modification of Zanjani (2002)

Zanjani (2002) shows¹⁶ that if there are significant costs associated with holding the capital necessary to pay for catastrophic losses (such as taxes, asymmetric information, warehousing), then the marginal price of insurance can be written as

$$p_i = \frac{\partial c_n}{\partial y_i} + \mu_i - \frac{dE(D_n)/dy_i}{1+r_f} + \delta \frac{\partial \hat{R}_n}{\partial y_i},$$

with the R_n function being the cost of capital for firm n , $E(D_n)$ being firm n 's implicit benefit from being allowed to default (the value of the default put option for firm n) and y_i the demand for the product in line i . Parameters μ_i and r_f are respectively the expected loss per unit of insurance and the risk free return; both are independent of who the insurer/reinsurer is. In our setting, we will posit $\frac{\partial c_n}{\partial y_i}$ in the

Zanjani pricing equation to be a constant so that we can write that $b_n = \frac{\partial c_n}{\partial y_i} + \mu_i$ is the value of the

intercept in the marginal cost function $C'_n(L)=b_n+k_nL$. Regarding the second part of the Zanjani marginal pricing equation, we will let the marginal price of insurance increase linearly with the size of the loss. In

other words, we define for some entity n (insurer n) $h_n(L) = -\frac{dE(D_n)/dy_i}{1+r_f} + \delta \frac{\partial \hat{R}_n}{\partial y_i}$ to be an

increasing linear function of L such that $h_n(L)=k_nL$, $h'_n(L)=k_n>0$ and $h_n(0)=0$ (i.e, there is no capital cost for assuming no risk). As Zanjani shows,¹⁷

$$R_n = \frac{1}{x} \left[\frac{\text{cov}(D_n, r_m) - \text{cov}(L, r_m)}{\text{var}(r_m)} \right] \frac{E(r_m) - r_f}{1+r_f}$$

¹⁶ Equation 17 in Zanjani (2002).

¹⁷ Equation 24 in Zanjani (2002).

$$\text{with } x = \delta - \frac{f + \tau r_f}{(1 + r_f)(1 - \tau)} > 0 \text{ and}$$

$$D_n = \max[0, L - E(L) - S_n(1 + r_f)] = \max[E(L) + S_n(1 + r_f), L] - E(L) - S_n(1 + r_f),$$

where D_n is the portion of claims not paid when insurer n defaults, S_n is the insurer's initial surplus and δ , f , τ and r_f are parameters that represent the discounted cost of holding capital, the frictional costs (underwriting, liquidity, etc.), the corporate tax rate and the risk free return respectively. Letting f and τ

be small, a necessary condition for $h'_n(L) > 0$ is $\frac{\partial^2 R_n}{\partial L^2} > 0$. This means that $h'_n(L) > 0$ if

$$\frac{\partial^2 \text{cov}(D_n, r_m)}{\partial L^2} > \frac{\partial^2 \text{cov}(L, r_m)}{\partial L^2},$$

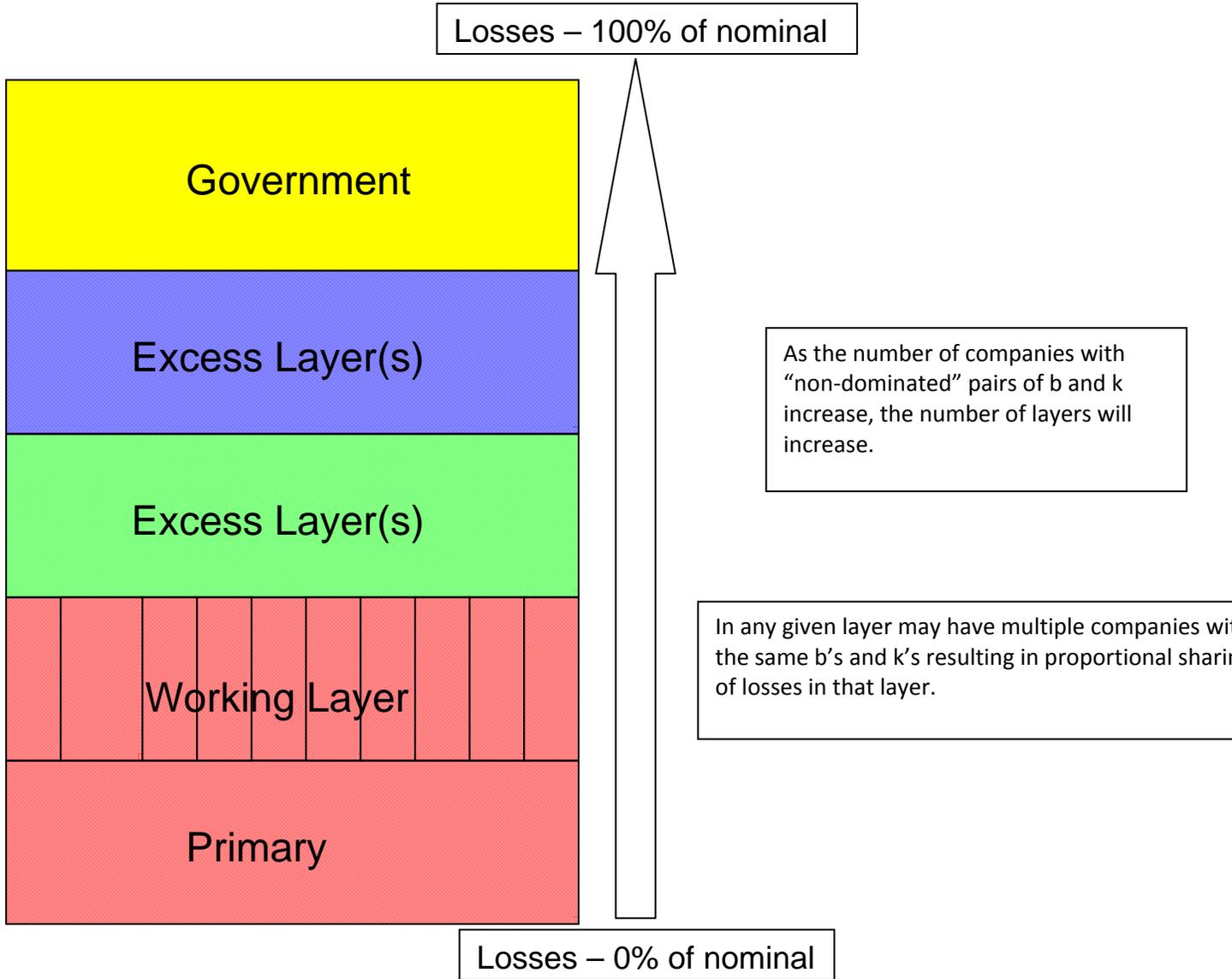
which we can presume it is since the payoff of an option is more

convex than that of the underlying security. For simplicity, let us focus on the linear case. We will thus set

$$k_n = \frac{1}{x} \left(\frac{E(r_m) - r_f}{1 + r_f} \right) = \frac{(E(r_m) - r_f)(1 - \tau)}{(1 + r_f)(1 - \tau)\delta - f - \tau r_f}.$$

This marginal price of insurance for a given new risk that needs to be insured is given by.

6.2 APPENDIX B: Insurance Program Prospective of Risk Sharing



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